Appendix D

Problems

D1 Introduction to the problems

The problems posed in this section are designed to develop facility in selecting material, shape and process, and in locating data. The first in each section are very easy; some of those which come later are more difficult. Difficulty, when it arises, is not caused by mathematical complexity — the maths involved is simple throughout; it arises from the need to think clearly about the objectives, the constraints, and the free variables. The level of this kind of difficulty is indicated by the number of daggers: † means easy, ††††† means hard. Many of the problems can be tackled by drawing simple bounds (vertical and horizontal lines) onto the charts or by using the material indices listed in Tables 5.7 on page 78 and in Table B1–B7, pages 408–412. Others require the derivation of new material indices; here the catalogue of Appendix A will help. The problems of Section D2 introduce the use of the charts. Sections D3, D4 and D5 explore the way in which indices, some including shape, are used to optimize the selection. Most of the problems use the hand-drawn charts which come with this book. A few require charts which are not part of the hand-drawn set. For these the appropriate output of the *CMS* software (which allows charts with almost any axes to be created at will) is given.

Process selection problems are given in Section D6, which requires the use of the Process Charts of Chapter 11. Section D7 contains data-search problems. Ideally, the use of handbooks should be combined here with the use of computer databases (for information on these, see Chapter 13, Section 13.8). The *CMS* database, particularly, is helpful here. The final problems of Section D8 illustrate the interaction between material properties and scale, and the optimization of properties for a given application.

And a final remark: any one of the Case studies of Chapters 6, 8, 10, 12 or 14 can be recast as a problem, either by giving the design requirements and appropriate material limits and indices, and asking for a selection to be made, or by asking that the design requirements be formulated and limits and indices derived. The Case studies themselves then provide worked solutions.

The best way to use the charts which are a feature of the book is to have a clean copy on which you can draw, try out alternative selection criteria, write comments, and so forth; and presenting the conclusion of a selection exercise is, often, most easily done in the same way. Although the book itself is copyrighted, the reader is authorized to make copies of the charts, and to reproduce these, with proper reference to their source, as he or she wishes.

D2 Use of materials selection charts

A component is at present made from brass (a copper alloy). Use Chart 1 to suggest two other metals which, in the same shape, would be stiffer. (†)

Use Chart 1 to find the material with modulus $E > 200\,\text{GPa}$ and density $\rho < 2\,\text{Mg/m}^3$. (†)

D2.3 Use the Modulus–Density Chart (Chart 1) to identify the subset of materials with both modulus $E > 100\,\text{GPa}$ and the material index

$$M = E^{1/3}/\rho > 2.15(\text{GPa})^{1/3}/(\text{Mg/m}^3).$$

where ρ is the density. (Remember that, on taking logs, this equation becomes

$$\log(E) = 3\log(\rho) + 3\log(M)$$

and that this plots as a line of slope 3 on the $\log(E)$ vs. $\log(\rho)$ chart, passing through the point $\rho = 1/2.15 = 0.46$ at $E = 1$ in the units of Chart 1.) (††)

D2.4 Which have the higher specific strength, σ_f/ρ: titanium alloys or tungsten alloys? Use Chart 2 to decide. (†)

D2.5 The bubble labelled 'WOOD PRODUCTS' on the Charts 1 and 2 refers to plywood, fibreboard and chipboard. Do these materials have a higher or a lower specific stiffness, E/ρ, than nylons? (†)

D2.6 Are the fracture toughnesses, K_{Ic}, of common engineering polymers like PMMA (perspex) higher or lower than those of engineering ceramics like alumina? Chart 6 will help. (†)

D2.7 The elastic deflection at fracture when an elastic-brittle solid is loaded is related to the strain-at-failure by

$$\varepsilon_{fr} = \frac{\sigma_{fr}}{E}$$

where E is Young's modulus and σ_{fr} is the stress which causes a crack to propagate:

$$\sigma_{fr} \approx \frac{K_{Ic}}{\sqrt{\pi c}}$$

Here K_{Ic} is the fracture toughness and c the length of the longest crack the material may contain. Thus

$$\varepsilon_{fr} = \frac{1}{\sqrt{\pi c}}\left(\frac{K_{Ic}}{E}\right)$$

The materials which, for a given crack-length c show the largest deflection at fracture are those with the greatest value of the material index

$$M = \frac{K_{Ic}}{E} \qquad \text{(D1)}$$

Use Chart 6 to identify three 'brittle materials' with exceptionally large fracture strains. (††)

D2.8 One criterion for design of a safe pressure vessel is that it should leak before it breaks: the leak can be detected and the pressure released. This is achieved by designing the vessel to tolerate a crack of length equal to the thickness t of the pressure vessel wall, without failing by fast fracture. The pressure p given by this design criterion is

$$p \le \left(\frac{K_{Ic}^2}{\sigma_y}\right)\left(\frac{1}{2RS}\right)$$

where σ_y is the yield strength (for metals, the same as σ_f), K_{Ic} is the fracture toughness, R is the vessel radius and D is a safety factor. The pressure is maximized by choosing the material with the greatest value of

$$M = \frac{K_{Ic}^2}{\sigma_y} \qquad \text{(D2)}$$

Use Chart 7 to identify three alloys which have particularly high values of M. Comment on their relative materials. (††)

D2.9 An engine test-frame requires a material which is both stiff (modulus $E > 40$ GPa) and has a high damping. Damping is the ability of a material to dissipate elastic energy: vibration-deadening materials have high damping. It is measured by the loss coefficient, η. Use Chart 8 to identify a subset of 4 possible materials for the engine test-frame. Comment on their suitability. (†)

D2.10 Use Chart 8 to identify material which should make good bells. (†)

D2.11 Use Chart 9 to identify a small subset of materials with the lowest thermal conductivity (best for long-term insulation), and to identify a small subset with the lowest thermal diffusivity (best for short-term insulation). (†)

D2.12 Use Chart 9 to find two materials which conduct heat better than copper. (†)

D2.13 The window through which the beam emerges from a high-powered laser must obviously be transparent to light. Even then, some of the energy of the beam is absorbed in the window and can cause it to heat and crack. This problem is minimized by choosing a window material with a high thermal conductivity λ (to conduct the heat away) and a low expansion coefficient α (to reduce thermal strains), that is, by seeking a window material with a high value of

$$M = \lambda/\alpha \qquad \text{(C3)}$$

Use Chart 10 to identify the best material for an ultra-high powered laser window. (Don't be surprised at the outcome — lasers really are made with such windows.) (††)

D2.14 Use Chart 12 to decide whether steels are more or less resistant than cast irons to thermal shock. (†)

D2.15 Table 5.7 tells us that the cheapest material for a column which will not buckle under a given axial load is that with the greatest value of the material index

$$M = \frac{E^{1/2}}{C_m \rho} \qquad \text{(C4)}$$

where E is the modulus, ρ the density and C_m the cost per kilogram of the material. Use Chart 14 to identify a subset of six materials that perform best by this criterion. How do these compare with the materials used in the construction of buildings? (Remember that, on taking logs, equation (C4) becomes

$$\log(E) = 2\log(C_m \rho) + 2\log(M)$$

and that this plots as a line of slope 2 on the $\log(E)$ vs $\log(C_m \rho)$ chart.) (††)

D2.16 Use Chart 16 to decide whether engineering ceramics are more or less wear-resistant than metals. (Remember that wear-rate, at a given bearing pressure, is measured by the wear-rate constant, k_a; low k_a means low wear rate.) (†)

D2.17 Use an index selected from Table B1, page 408, together with Chart 17, to determine whether the energy-content of a reinforced-concrete panel of prescribed stiffness is greater or less than that of a wood panel of the same stiffness. Treat the panel as a plate loaded in bending. (††)

D2.18 A beam-like component of specified section shape, designed to carry a prescribed bending load without failing, could be moulded from nylon or die-cast from a zinc alloy. Select the appropriate index from Appendix B, Table B1.1 and use Chart 18 to decide which has the lower energy content. (††)

D2.19 A material is required for the blade of a rotary lawn-mower. Cost is a consideration. For safety reasons, the designer specified a minimum fracture toughness for the blade: it is $K_{Ic} > 30\,\mathrm{MPa\,m^{1/2}}$. The other mechanical requirement is for high hardness, H, to minimize blade wear. Hardness, in applications like this one, is related to strength:

$$H \propto 3\sigma_f$$

where σ_f is the strength (Chapter 4 gives a fuller definition). Use Chart 7 to identify three materials which have $K_{Ic} > 30\,\mathrm{MPa\,m^{1/2}}$ and the highest possible strength. To do this, position a 'K_{Ic}' selection line at $30\,\mathrm{MPa\,m^{1/2}}$ and then adjust a 'strength' selection line such that it just admits three candidates. Transfer this strength-limit to Chart 15, and use it to rank your selection by material cost; hence make a final selection. (††)

D3 Deriving and using material indices

D3.1 Material indices for elastic beams in bending

Start each of the four parts of this problem by listing the function, the objective and the constraints.

(a) Show that the best material for a cantilever beam of given length ℓ and given (i.e. fixed) square cross-section ($t \times t$) which will deflect least under a given end load F (Appendix A, Section A3) is that with the largest value of the index $M = E$, where E is Young's modulus (neglect self-weight).

(b) Show that the best material choice for a cantilever beam of given length ℓ and with a given section ($t \times t$) which will deflect least under its own self-weight is that with the largest value of $M = E/\rho$, where ρ is the density.

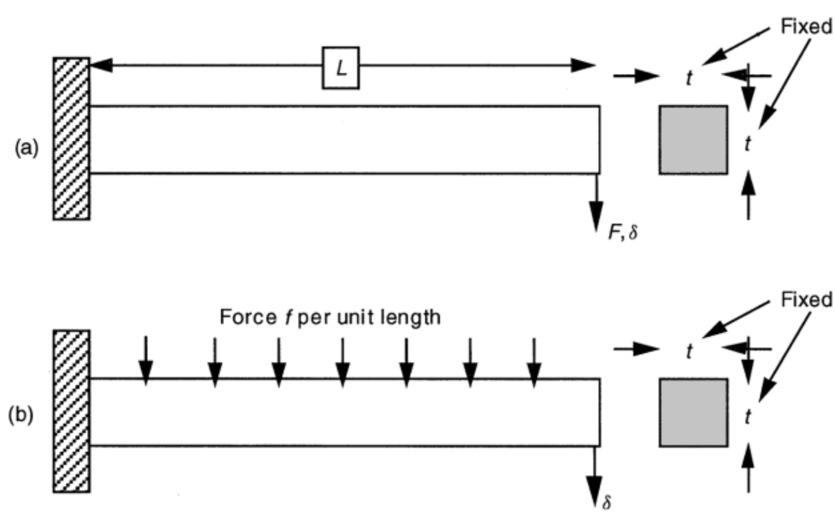


Fig. D3.1 Beams loaded in bending, with alternative constraints.

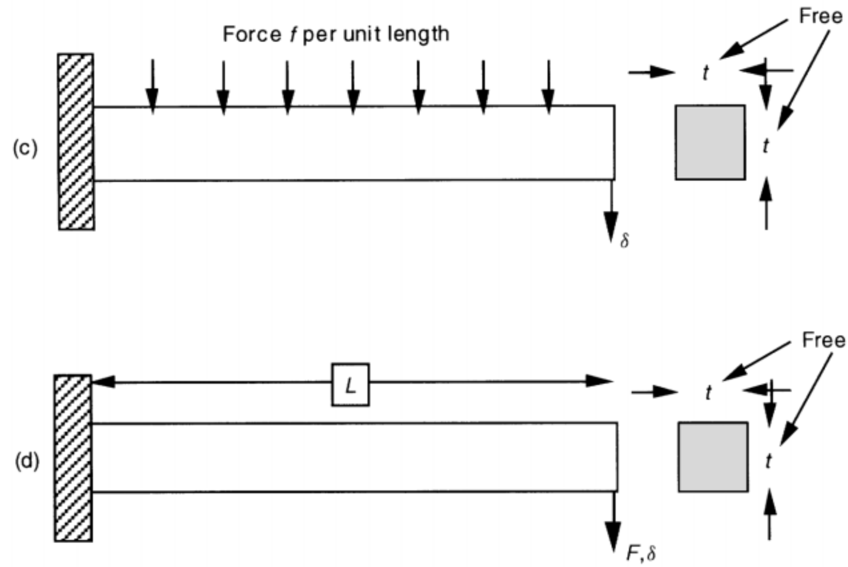


Fig. D3.1 (*continued*)

(c) Show that the material index for the lightest cantilever beam of length ℓ and square section (not given, i.e. free) which will not deflect by more than δ under its own weight is $M = E/\rho^2$.

(d) Show that the lightest cantilever beam of length ℓ and square section (free) which will not deflect by more than δ under an end load F is that made of the material with the largest value of $M = E^{1/2}/\rho$ (neglect self weight). (†††)

D3.2 Effect of geometric constraints on material indices

Derive

(a) equation (5.12) of the text, and
(b) equation (5.13) of the text, using the method in the section which precedes the two equations. (†††)

D3.3 The health service crutch

You are asked to re-design the Health Service Crutch. It is designed for a person with only one serviceable leg, and is, in mechanical terms, a slender column with a padded top (which goes under the armpit) loaded in compression by a known maximum load F. The crutch should be as light as possible. It would seem desirable that it should not fail by elastic buckling. Derive a material index for the lightest column of solid circular section, which will not buckle elastically under an end-load, F.

Write, first, an equation for the weight of the crutch; then one for failure by elastic buckling — Euler's law (Appendix A, Section A5) — and use it to eliminate the free variable — the radius — from the first equation.

Use the result to select a subset of five candidate materials for the crutch, using the appropriate Materials Selection Chart. (†††)

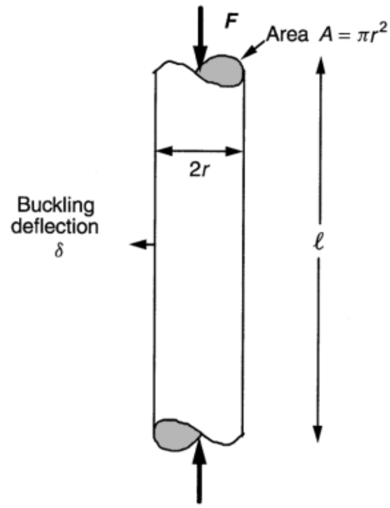


Fig. D3.2 The crutch.

D3.4 The daisy-wheel print head

A daisy-wheel print head (Figure D3.3) is a set of elastic fingers, each carrying a character-face at its tip. The tip is struck from behind by a hammer which compresses it against the paper. Unlike the golf ball of Case study 6.8, only a tiny part of the wheel is loaded (and that in compression), allowing lighter construction: a typical golf-ball weighs 12 g; a typical daisy-wheel, only 8. But the material requirements are different. A golf-ball is positively positioned, lifted from the paper by a spring; a daisy-wheel leaves the paper only because of the elastic recovery of the finger carrying the type. This recovery time is related to the stiffness of the finger, which we model as a cantilever (Appendix A, Section A3) of length ℓ, width b, and thickness t. We wish to minimize the mass m of the finger (to allow rapid repositioning) where

$$m = b\ell t\rho$$

subject to the constraint that the finger is adequately stiff.

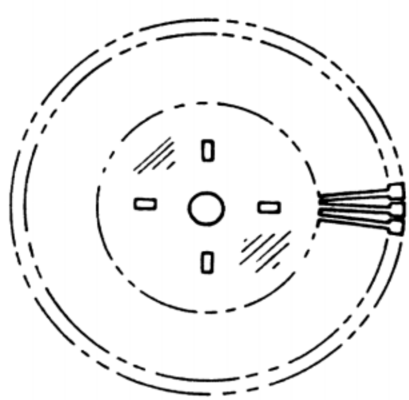


Fig. D3.3 A daisy-wheel print head.

Treat ℓ and b as fixed by the design (they are certainly severely constrained by it), leaving t as the free variable. Derive a material index for selecting materials which meet this stiffness constraint at minimum mass. Use it to identify five polymers which might be suitable for making daisy-wheel print heads. (†††)

D3.5 Materials for vaulting poles

You are hired by a pole vault enthusiast who wishes to equip himself with the very best of vaulting poles. International standards set the length and maximum section of the pole, which must be cylindrical. In use, the pole bends elastically, storing energy which is released at the top of the flight path, projecting the jumper over the bar (Figure D3.4). Assume that the best pole is that made of the material which stores (and then releases) the most elastic energy per unit volume without failing.

Derive a material index and use it to identify a subset of materials which should make good vaulting poles. Are the results consistent with what you know about real vaulting poles? How does the selection change if the criterion is that of storing the most elastic energy per unit weight, instead of volume? (†††)

D3.6 Springs for trucks

In vehicle suspension design is it desirable to minimize the mass of all components. You have been asked to select a material and dimensions for a light-weight spring to replace the steel leaf-spring of an existing truck suspension.

The existing leaf-spring is a beam, shown schematically in Figure D3.5. The new spring must have the same length L and stiffness S as the existing one, and must deflect through a maximum safe displacement δ_{max} without failure. The width b and thickness t are not constrained.

Derive a material index for the selection of a material for this application. Note that this is a problem with two free variables: b and t; and there are two constraints, one on safe deflection δ_{max} and the other on stiffness S. Use the two constraints to fix the two free variables. (†††)

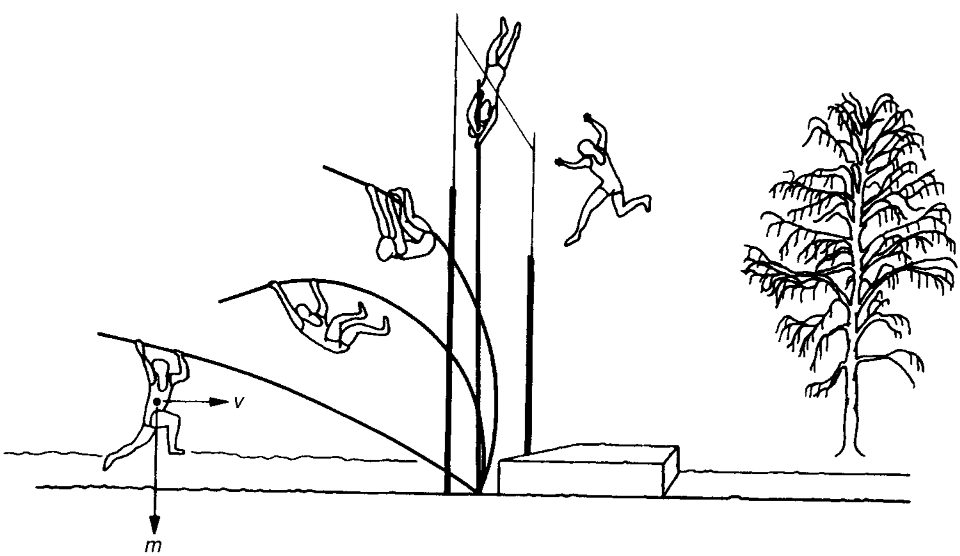


Fig. D3.4 A pole-vaulter.

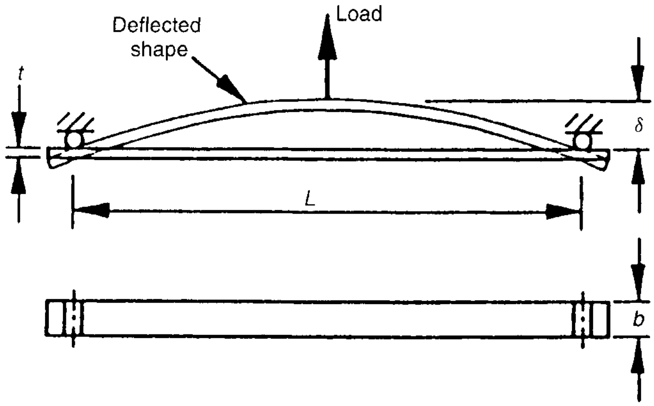


Fig. D3.5 A truck spring. It must have a given stiffness, S, and be capable of deflection through δ_{max} without failure. The objective is to minimize the mass m.

D3.7 Elastic con-rods

It has been suggested that the bearing and gudgeon pin which form the little-end bearing of the con-rod/piston assembly of a reciprocating engine or pump could be replaced by an elastic ligament or hinge, as shown in Figure D3.6.

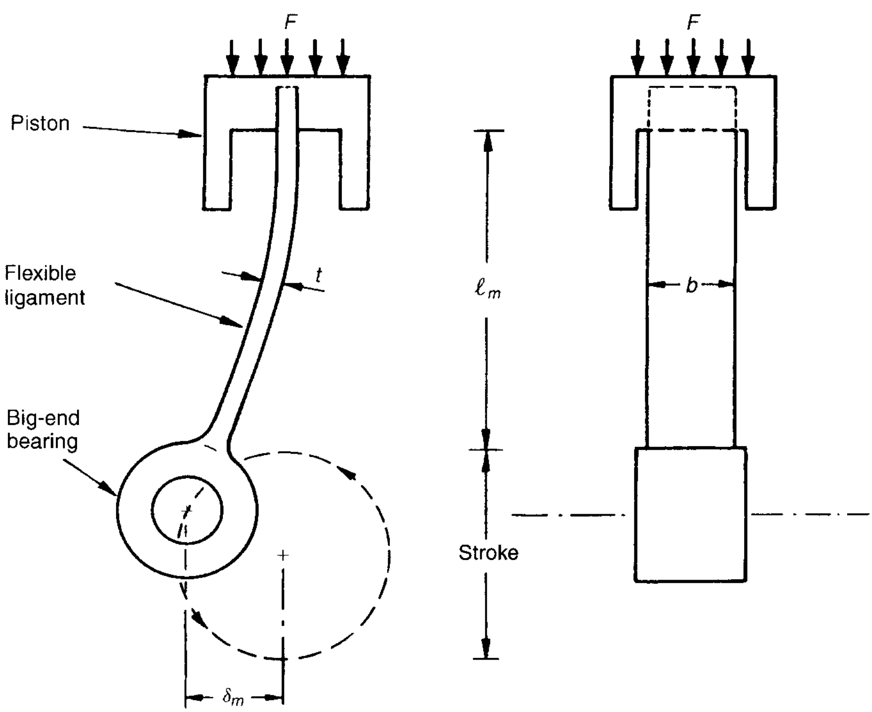


Fig. D3.6 The elastic connecting rod. The flexible ligament is rigidly fixed to the piston.

(a) Discuss the mechanics aspects of this suggestion, assuming that the objective is to maximize the allowable thrust, F. Formulate the constraints which must be met by such a hinge.
(b) Assume the width, b, of the hinge is set by clearances within the device (i.e. it is fixed) and that its length ℓ_m and the maximum lateral deflection δ_{max} (equal to half the stroke) are both specified. Derive a material index for the hinge and use it to identify materials which allow large F without causing the elastic ligament or buckle or fail plastically.

Proceed as follows

(i) Write down an expression for the buckling load of the ligament (see Appendix A, Section A5 for the relevant equations). Use this to identify the minimum thickness t of the ligament which will not buckle under an axial load F.
(ii) Write down an equation for the maximum stress σ in the bending ligament. It is the sum of two contributions, one from the axial load F, the other from the elastic bending (Appendix A, Section A4) which is greatest when the deflection is δ_{max}. The design goal is to maximize F without causing the ligament to fail, which it will of the stress in it anywhere exceeds its failure stress, σ_f. (Fatigue can be allowed for by applying a sufficiently large safety factor, S).
(iii) Substitute for t from (i) into this result, giving a (quadratic) equation for F.
(iv) Now identify the combination of material properties which maximize F. Examine the combination of material properties that appear in these limits: they are the relevant material indices. Use them to identify a small subset of materials which allow high F_{max}. Use common sense to reject any which, for other reasons, are obviously inappropriate, identifying the constraint which causes them to be so. (†††††)

D3.8 The pipeline inspection gadget

A PIG, to someone in the oil and gas business, is not an animal; it is a Pipeline Inspection Gadget (Figure D3.7). PIGs are pulled through pipelines to detect and remove obstructions. Intelligent PIGs do more: they probe the condition of the pipe wall and its surroundings, by vibrational methods, looking for defects. The body of the PIG is a cylindrical shell with an outer diameter fixed by the pipe through which it must pass. The body should be as light as possible, but have natural resonance frequencies which are high so that they do not interfere with the low frequency vibrations used to probe the condition of the pipe itself. Identify a subset of materials from which the PIG body might be made which minimize weight while keeping all natural frequencies greater than a given value ω^*.

Proceed by writing an equation for the mass of the cylinder of length ℓ and wall-thickness t (ignore the end caps). The natural frequencies of the longitudinal flexural modes (Appendix A, Section A12) of the shell are

$$\omega_j = \left(\frac{EI}{m_o}\right)^{1/2}\left(\frac{j\pi}{\ell}\right)^2$$

where $m_o = \rho A$ is the mass per unit length and $j(= 1, 2, 3\ldots)$ is the mode number. The second moment of area for this mode is

$$I_1 = \pi R^3 t$$

and the appropriate area A_1 is

$$A_1 = 2\pi R t$$

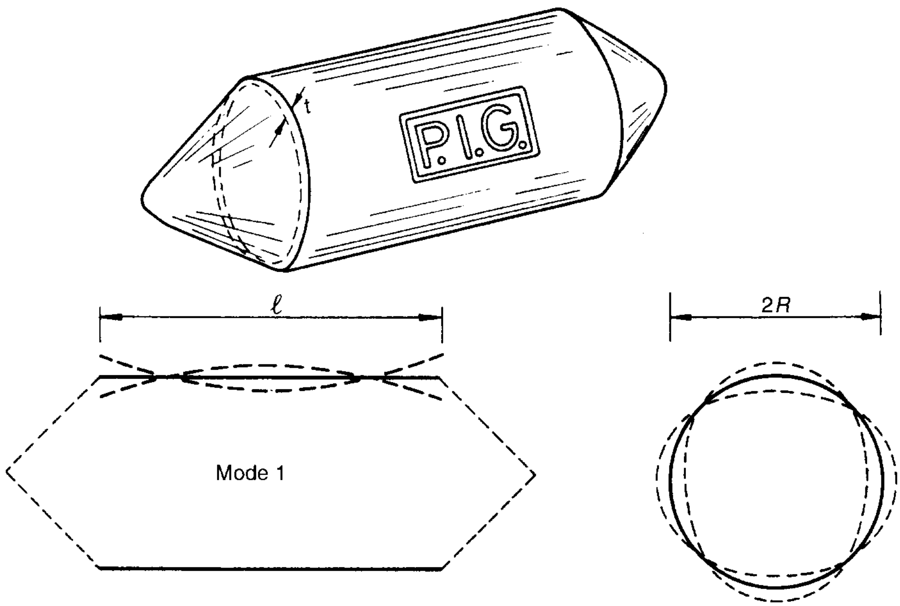


Fig. D3.7 A pipeline inspection gadget. The lowest natural frequencies of its shell must be kept high while at the same time minimizing its mass.

where t is the wall thickness of the shell. The natural frequencies of the circumferential modes (Appendix A, Section A12) are proportional to ω_j again, but with

$$I_2 = \frac{\ell t^3}{12}$$

and

$$A_2 = \ell t$$

Use these to find the lowest natural frequency and use the result to eliminate the free variable, t. Hence derive a material index and use it to make a selection. (†††)

D3.9 The tape-deck opening spring

A spring-loaded device is required to open the tape-deck cover of a compact tape-player when the catch is released. Describe the steps you would follow in proceeding from this 'market need' to the final production of the device, with particular reference to the source and types of material data you would require at each stage of the design process. Your description should include an analysis of the requirement, and some possible mechanical solutions.

A proposed design for the device is sketched in Figure D3.8. It comprises a torsion bar with a rigid actuator arm, and the following features of the design are specified:

(i) The length of the arm, H;
(ii) The range of movement of the end of the arm, δ_{max};
(iii) The opening force F when the lid is shut.

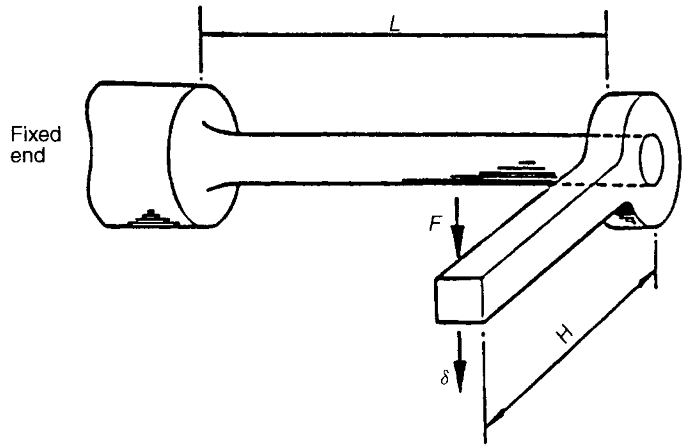


Fig. D3.8 A torsion spring to open a tape-deck lid.

It is required to choose a suitable material for the torsion bar in order to minimize (a) its length (b) its radius and (c) its volume. Find the appropriate groupings of material properties, and draw lines on the relevant Materials Selector Chart to indicate the likely candidate materials. Eliminating materials which have any obvious drawbacks, select the most appropriate material, and write down expressions for the dimensions of the bar in terms of the design specifications. (†††)

D3.10 Material to resist thermal shock

When the temperature of the surroundings of an engineering component changes suddenly from T_o to a lower one T_1, the temperature gradient within it generates stresses, tensile at the surface, which can be damaging. If they exceed the brittle fracture strength σ_f, cracks are nucleated and chunks may spall off. If, instead, they exceed the yield strength, plastic flow takes place and — if repeated — may cause fatigue failure.

Consider a material which is suddenly cooled from T_o to T_1. A thin surface skin, now at T_1, tends to contract but is strongly bonded to the bulk of the component, which is still at T_o. The thin skin is constrained by the bulk, and is therefore elastically strained by an amount which is equal and opposite to the thermal strain. The elastic strain gives (via Hooke's law) a stress. Use this model as a basis for deriving an equation for the thermal stress and equate this to the failure stress to give an equation for the maximum temperature interval which the component can sustain without failure. Read off the combination of properties (the material index) which maximizes this interval. (††)

D3.11 Bearings for bicycles

You are employed by a company which makes bicycles. To counter an influx of cheap imported bicycles from some remote country where the climate is warm and wages are low, the company decides to redesign its standard bike to make it as cheap as possible. You are asked to identify the cheapest material for the bearings. Ball bearings are out; what is wanted is the cheapest material which will give adequate wear resistance when used as a sleeve bearing.

Suggest materials for cheap bicycle bearings. The simplest shape is a sleeve bearing: a thin-walled cylinder of length ℓ, radius r, and wall-thickness t all three of which are fixed by the need to match

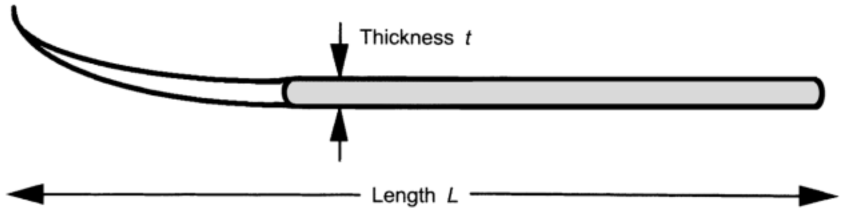


Fig. D3.9 Schematic of a disposable fork.

standard shaft and pedal dimensions. Use Chart 16 to identify possible candidates for the bearing and rank them by cost, using approximate data from Chart 14. Comment on the selection you make. Need for lubrication? Corrosion problems which might greatly increase the wear rate? Does your final conclusion match with your experience of cheap bearings? (††)

D3.12 Disposable knives and forks

Disposable knives and forks are ordered by an environmentally-conscious pizza-house. The shape of each (and thus the length, width and profile) are fixed, but the thickness is free: it is chosen to give enough bending-stiffness to cut and impale the pizza without excessive flexure. The pizzeria-proprietor wishes to enhance the greenness of his image by minimizing the energy-content of his throw-away tableware, which could be moulded from polystyrene (PS) or stamped from aluminium sheet.

Establish an appropriate material index for selecting materials for energy-economic cutlery. Model the eating-implement (Figure D3.9) as a beam of fixed length L and width w, but with a thickness t which is free, loaded in bending. The objective-function is the energy content: the volume times the energy content, q, per unit volume. The limit on flexure imposes a stiffness constraint (Appendix A Appendix A, Section A3). Use this information to develop the index, and use it, with Chart 17, to decide between polystyrene and aluminium.

Flexure, in cutlery, is an inconvenience. Failure — whether by plastic deformation or by fracture — is more serious: it causes loss-of-function; it might even cause hunger. Repeat the analysis, using a strength constraint (Appendix A, Appendix A, Section 4). Use Chart 18 to decide if your initial choice of material is still valid. (†††)

D4 Selection with multiple constraints

The four problems of this section illustrate the application of multiple criteria. All four are solved by using the methods of Chapter 9.

D4.1 A light, stiff, strong tie

A tie, of length ℓ, loaded in tension, is to support a load F, at minimum weight, without failing or extending elastically by more than δ (two constraints). Follow the method of Chapter 9 to establish two performance equations for the mass, one for each constraint, from which two material indices and one coupling equation which links them are derived.

Use these and Chart 5 to identify candidate materials for the tie (a) when $\ell/\delta = 100$ and (b) when $\ell/\delta = 10^3$. Ignore ceramics in your selection. They have great strength in compression but are brittle and, if flawed, have low strength in tension. (††††)

D4.2 A cheap column that must not buckle or crush

The best choice of material for a light strong column depends on its aspect ratio: the ratio of its height h to its diameter d. This is because short, fat columns fail by crushing; tall slender columns buckle instead. Derive two performance equations for the material cost of a column of solid circular section, designed to support a load F, one using the constraints that the column must not crush, the other that it must not buckle.

Some possible candidates for the column are listed in Table D4.1. Use these to identify candidate materials (a) when $F = 10^5$ N and $h = 2$ m; and (b) when $F = 10^3$ N and $h = 20$ m. Ceramics are admissible here, because they have high strength in compression. Reject any candidates which are expensive. (††††)

D4.3 A light, safe, pressure vessel

When the pressure vessel has to be mobile, its weight becomes important. Aircraft bodies, rocket casings and liquid-natural gas containers are examples; they must be light, and at the same time they must be safe. What are the best materials for their construction?

Write, first, a performance equation for the mass m of the pressure vessel. Assume, for simplicity, that it is spherical, of specified radius R, and that the wall thickness, t (the free variable) is small compared with R. The pressure difference, p, across this wall is fixed by the design. The first constraint is that the vessel should not yield — that is, that the tensile stress in the wall should not exceed σ_f. The second is that it should not fail by fast fracture; this requires that the wall-stress be less than $K_{Ic}\sqrt{\pi c}$, where c is the length of the longest crack that the wall might contain. Use each of these in turn to eliminate t in the objective equation; use the results to identify two material indices and a coupling relation between them: it contains the crack length, c.

Figure D4.1 shows the output of the CMS software with the two material indices as axes. Plot the coupling equation onto this figure for two values of c: one of 6 mm, the other of 1 µm. Identify candidate materials for the vessel for each case. (††††)

D4.4 The dare-devil plank

You are seized by the noble notion of raising money for charity by traversing a narrow street on a plank resting on the roofs of the houses on either side, a distance ℓ apart. You feel some concern, first, that the plank is strong enough to support you without breaking (a *strength* constraint), second,

Table D4.1 Data for candidate materials for the column

Material	*Density* ρ *(kg/m³)*	*Cost/kg* C_m *(£/kg)*	*Modulus* *E (GPa)*	*Compression strength* σ_f *(MPa)*
Wood (spruce)	490	0.4	15	45
Brick (common, hard)	2100	0.35	25	120
Sandstone	2400	0.4	50	130
Granite	2600	0.6	80	450
Poured concrete	2300	15	15	0.1
CFRP (laminate)	1600	35	100	450
GFRP (laminate)	1780	4,5	28	300
Structural steel	7850	0.4	210	1100
Al-alloy 6061	2700	1.2	69	130

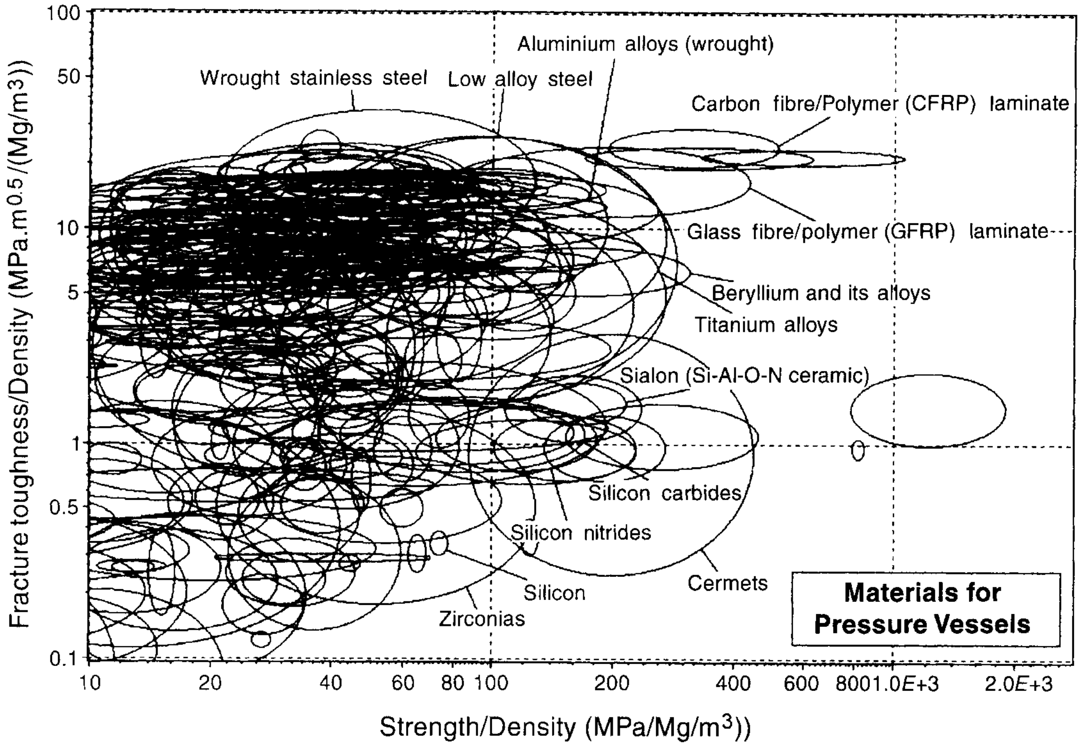


Fig. D4.1 Part of a computer-generated Selection Chart for specific fracture toughness, K_{Ic}/ρ and specific strength, σ_f/ρ.

that it will not sag too much when you are in the middle (a *stiffness* constraint), and, finally, that it be as light as possible so that you can get it up on to the roof by yourself (*objective function* for mass, to be minimized). The plank has a solid rectangular section of thickness t (free) and width b (fixed by your wish for a good footing). Derive two material indices for material selection based on the specification given above. This is a problem with one free variable (t) and two constraints (strength and stiffness). Proceed as follows.

(a) Write down an expression for the mass m of the plank; it is the objective function.
(b) Write down an expression for the failure load F of the plank in terms of its moment of area I, thickness t and its yield or fracture strength σ_f (Appendix A, Section A4); it must not be less than F^* (the force caused by your weight). Solve for t and substitute into the objective function, giving the first mass equation, m_1.
(c) Write down an expression for the deflection δ produced by a load F^* in terms of I and the modulus E (Appendix A, Section A3); it must not be more than δ^* (the maximum acceptable deflection). Again solve for t and substitute into the objective function, giving the second mass equation, m_2.

Given that $\ell = 6$ m, $b = 100$ mm, your weight (times a prudent safety factor) is 100 kg and that a deflection exceeding $\delta^* = 100$ mm would make life difficult, make a selection of material for the beam form the candidates listed in Table D4.2, below. (††††)

Table D4.2 Potential candidate materials for the plank

Material	*Density* ρ *(kg/m*^{3}*)*	*Modulus* *E (GPa)*	*Strength* σ_f *(MPa)*
Wood (spruce)	490	15	45
CFRP (laminate)	1600	100	450
GFRP (laminate)	1780	28	300
High-strength steel	7850	210	1100
Al-alloy 6061	2700	69	130

D5 Selecting material and shape

The problems of the section involve the use of shape factors for their solution.

D5.1 Improvised ski-poles

You wish to improvise a ski-pole using standard stock. The pole must meet a stiffness specification and be as light as possible. Current stock includes:

(a) Wood poles in a range of solid circular sections.
(b) Aluminium tubing in a range of diameters with the ratio (tube radius/wall thickness) up to 15.
(c) Steel tubing in a range of diameters with the ratio (tube radius/wall thickness) up to 18.

Derive (or retrieve from Table 8.1, p. 195) a material index for selecting materials for a light stiff beam, allowing for shape. Compare the three candidate stock-materials using this index, tabulating the results. Plot the stock materials onto Chart 1, and draw any conclusions you can about other promising material-shape combinations for the ski-pole. Material properties are listed below.

Repeat the calculation, using the bending strength of the pole as the design constraint rather than its stiffness. (†††)

D5.2 Light tubular display stands

A concept for a lightweight display stand is shown in Figure D5.1. The frame must support a mass 100 kg (to be placed on its upper surface) at a height $h = 1$ m without failing by elastic buckling. It is to be made of stock tubing and must be as light as possible. Derive (or retrieve from Table 8.1, page 195) a material index for the tubular material of the stand which meets these requirements, and which includes the shape of the section, described by the shape factor

$$\phi_B^e = \frac{4\pi I}{A^2}$$

where I is the second moment of area and A is the section area. Tubing is available from stock in the following materials and sizes. Use this information and the material index to identify the best

Table D5.1 Materials for a light ski-pole

Material	*Density* ρ *(kg/m*^{3}*)*	*Modulus* *E (GPa)*	*Strength* σ_f *(MPa)*
Wood (spruce)	490	15	45
Cold drawn mild steel tube	7850	210	600
Cold drawn Al tube	2700	69	130

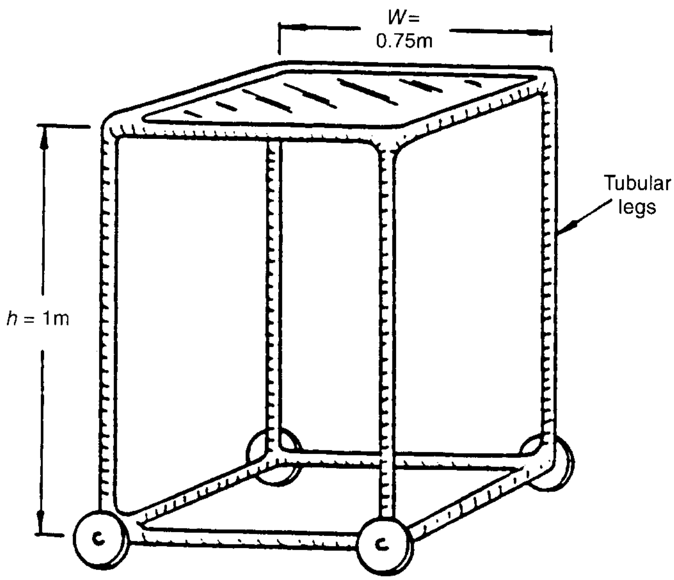


Fig. D5.1 A concept for a tubular display stand.

Table D5.2 Materials for the display stand

Material	*Tube radius*	*Wall thickness/ tube radius*
Aluminium alloys	All radii up to 25 mm	0.07 to 0.25
Steel	All radii up to 30 mm	0.045 to 0.1
Copper alloys	All radii up to 20 mm	0.075 to 0.1
Polycarbonate (PC)	All radii up to 10 mm	0.15 to 0.3
Various woods	All radii up to 40 mm	Solid circular sections only

stock material for the stand. Check that a stand made of your selection will actually support the design load. (††††)

D5.3 Energy-efficient floor joists

Section 8.4, page 200, of the text compared solid wood beams with shaped steel *I*-beams for use as floor joists in buildings on the basis of weight. When the design is stiffness-limited, the wooden joist with $\phi_B^e = 2$ and the shaped steel one with $\phi_B^e = 25$ performed equally well — that is, they had the same weight. But is the steel beam more energy-intensive than the wood?

(a) Locate from Table 8.1, page 195, (or derive if you prefer) the material index for shaped beams loaded in bending. Select that for stiffness-limited design at minimum energy content. Extract data from Chart 17 (p. 370) for modulus E and energy content per unit volume $q\rho$. Hence compare the value of the material index for the two beams.

(b) Answer the problem in a second way. Construct a line of slope 1, passing through 'steels' on Chart 17, and move steels down both axes by the factor 25, as in Figure 8.6, page 202. The shaped steel can be compared with other materials — wood, say — using the ordinary guide-lines which are shown on the figure. Is the conclusion the same?

(c) Repeat the two operations for strength-limited design for the two beams.
(d) What do you conclude about the relative energy-penalty of design with wood and with steel? (††††)

D5.4 Cheap floor joists

The Problem D5.3 required a comparison of wood as a solid beam with steel as an I-beam, on an energy-content basis. Compare the two, instead, on a material-cost basis, using Charts 14 and 15. (††††)

D5.5 An energy-absorbing bumper

The bumper for a military vehicle is to be designed on the basis of its ability to absorb energy in a mild collision without permanent deformation. For the purpose of analysis, the bumper is idealized as a thin-walled tube attached to the vehicle at its ends by rigid supports. The length and outer diameter of the bumper are fixed. Ignore contact stresses and any tendency to buckle at the point of impact.

(a) If the wall thickness of the tube is held constant, identify the combination of materials properties which determine the maximum energy the bumper can store without permanent damage. Use the appropriate design chart to select candidate materials for the bumper.
(b) If the wall thickness of the tube, though thin, can be varied, identify the combination of material properties which, for a given stored energy, will minimise the weight of the bumper. By combining information from two design charts, identify two candidate materials which would be sensible choices for the bumper.
(c) Discuss production methods appropriate to your selection, with particular reference to batch size and to the method of joining the bumper to the vehicle. Comment on how these consideration might influence your final selection for cases (a) and (b) above. (††††)

D5.6 Determining shape factors

A shape factor measures the gain in stiffness or in strength, relative to a solid circular cylinder, by shaping the material. Thus the shape factor for elastic bending if a beam is the ration of the bending stiffness of the shaped beam to that of a solid cylinder of the same length and mass. Shape factors ϕ can be determined analytically by calculating I and A for the section and using the equation

$$\phi_B^e = \frac{4\pi I}{A^2}$$

and the others like it. Alternatively, shape factors can be found by experiment, by measuring the stiffness S and mass m of a beam of length ℓ, made of a material with modulus E and density ρ, by inverting equation (7.26) of the text, (and the others like it) to give

$$\phi_B^e = \left(\frac{4\pi\ell^5 S_B}{C_1 m^2}\right)\left(\frac{\rho^2}{E}\right) \qquad \text{(D5.1)}$$

(a) Calculate analytically the shape factor for a thin-walled box girder with square section 40 mm × 40 mm and wall thickness 2 mm.
(Answer: $\phi_B^e = 10$.)

(b) Calculate the shape factor ϕ_B^e of an I-section beam with a bending stiffness 10^8 N/m if the bending stiffness of a solid cylinder of the same material, weight and length is 2×10^6 N/m.

(c) Calculate the shape factor ϕ_B^e from the following experimental data, measured on an aluminium alloy beam loaded in 3-point bending (for which $C_1 = 48$ — see Appendix A, Section A3) with:

Stiffness	$S_B = 7.2 \times 10^5$ N/m
Mass	$m = 1$ kg
Length	$\ell = 1$ m
Density	$\rho = 2670$ kg/m^3
Modulus	$E = 69$ GPa

(Answer $\phi_B^e = 20$.) (†††)

D5.7 Umbrella ribs

Umbrella design has a long history: sophisticated models can be seen in Chinese prints of the 9th century and before. The ribs of an umbrella (Figure D5.2) are loaded in bending; the struts are loaded in compression. The ribs must deflect elastically to take up the curved shape of the umbrella, exerting a specified restoring moment M on the fabric when the umbrella is fully open (thus the bending stiffness is specified). They must do this without failing by plastic collapse or fracture when this moment (or some multiple of it — to allow for mishandling) is applied. The best rib is that with the highest failure moment, M_f, which also meets the stiffness constraint.

(a) Derive a material index for selecting materials for ribs of solid circular section of an umbrella which meets these conditions. Note that the specified working moment M is a constraint; the objective function describes the failure moment, M_f, which is to be maximized. Use the appropriate Materials Selection Chart to identify a subset of materials which would perform well in this application. Comment on any problems of production, joining, cost, etc, associated with each choice.

(b) Umbrellas from the Far East have solid bamboo-wood ribs (they are cut from the solid wall of the hollow bamboo stalk). These ribs have a near-circular section. Bamboo wood has a modulus of 10.5 GPa and a failure stress of 127 MPa. Add this solid material to a copy of the Chart and hence make a judgement about its suitability for umbrella ribs.

(c) Calculate the weight-saving if the ribs were made out of hollow tubes of bamboo wood with a ratio of wall-thickness to diameter of 4:1. (††††)

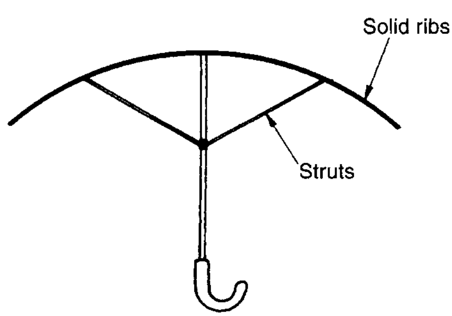
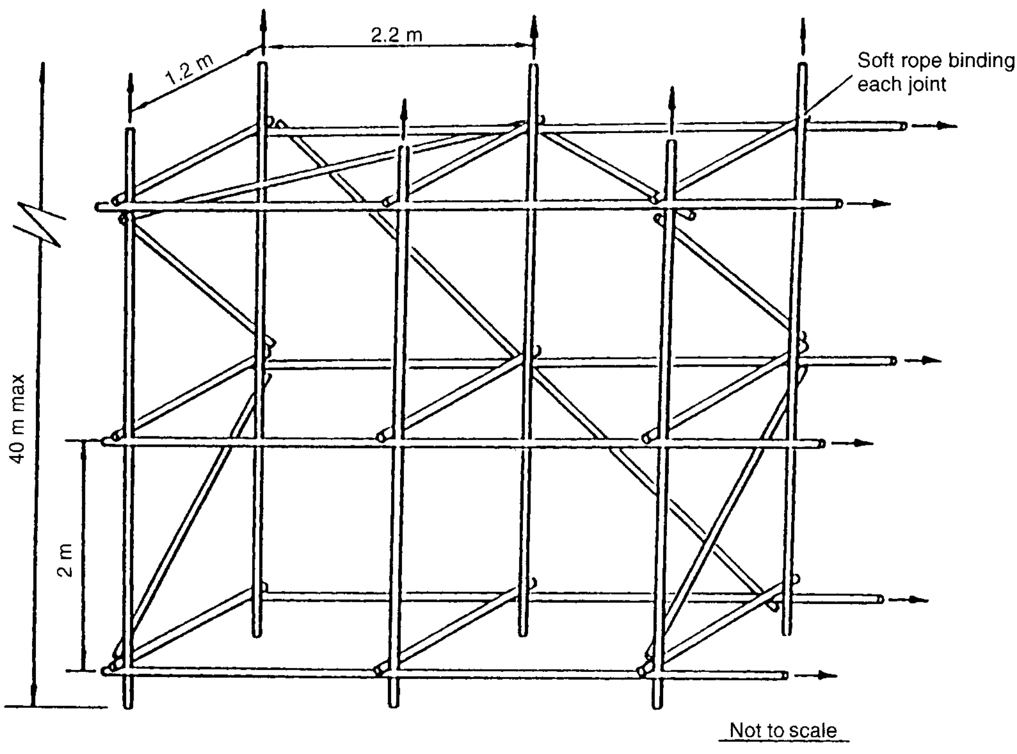


Fig. D5.2 An umbrella or contre-soleil, depending on the season.

D5.8 Bamboo scaffolding

Bamboo scaffolding has been used in China for building construction for over 2000 years. The bamboo lengths are tied together with soft rope. You have been commissioned to investigate and report on this with regard to modern technology, and to propose a concept for a modern scaffolding system to replace the indigenous one.

Figure D5.3 shows a schematic of the bamboo scaffolding, giving the basic dimensions. Note that there are vertical, short and long horizontal, and cross members. The figure also shows a



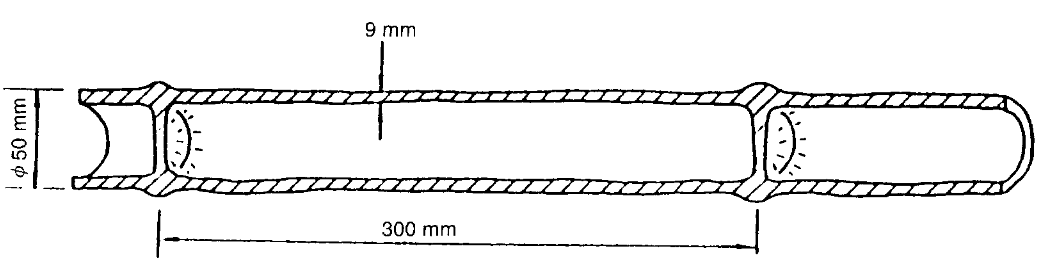


Fig. D5.3 Bamboo scaffolding (above). Bamboo is light, stiff and strong, partly because of its intrinsic structure and partly because of the shape of its section.

cross-section through a short length of typical scaffold bamboo. Bamboo scaffolding is used to a maximum height of 40 meters.

(a) Discuss the problems associated with each member type, describe its potential failure mode, and any problems associated with the fixing points.
(b) For the design of a new proposed system, outline the main design and materials selection steps, showing the feedback between the material choice, the section shape and the modes of loading.
(c) Select the best candidate material for each member, with respect to performance at minimum weight, then with respect to performance at minimum material cost, and finally, with respect to material energy content. Explain your choices.
(d) Discuss the problem of joining scaffolding members, recalling that rapid deployment and removal are essential. (†††)

D6 Selecting processes

D6.1 Making a shaker table

The shaker-table of Figure 6.31 is to be made of a magnesium alloy (melting point = 905 K; hardness = 800 MPa; density = 1.78 Mg/m^3). The diameter of the table is 2 m; the thickness of the table and its webs is 100 mm. The top surface and hub of the table are to be finished to a tolerance, T, of ± 0.07 mm and a RMS roughness, R, of 5 μm. The finish of the remaining surfaces is not critical. Suggest possible process routes. (Use the Size/Slenderness chart, Figure 11.31, and the Tolerance/Roughness chart, Figure 11.33. or the *CPS* software if available.) (†††)

D6.2 Forming a fan

A small fan mixes fuel–air mixture as it enters the burner-can of a high performance aircraft gas turbine. The fan runs at 650°C and is at present made of a stainless steel (density = 7.9 Mg/m^3). It is proposed to reduce the weight of the fan (to allow it to accelerate more quickly) by material substitution. In particular, the fan (it is proposed) might be made of boron carbide (melting point = 2620 K; hardness = 24 GPa; density = 2.5 Mg/m^3), without change of shape.

The fan has a simple shape: a disk, 100 mm in diameter and 5 mm thick, carries 8 radial fins attached to one of its faces; the fins are 5 mm thick and project 15 mm normal to the face. The tolerance on disk and fins is ± 0.3 mm, the RMS surface roughness should be less than 3 μm. Use the process charts (or the *CPS* software if available) to identify possible ways of shaping the fan. (†††)

D6.3 A computer case

A case is required for a notebook computer. The sales department insists on an A4 footprint, and a thickness no greater than that of a paperback novel. Translated into more rational units, the outer dimensions of the case are 280 × 220 × 20 mm, with a wall thickness not exceeding 2 mm. It is to be made in two pieces (a base and a lid, each about the same size) from a tough thermoplastic. The tolerance T on the larger dimensions is specified as ± 0.5 mm; the RMS roughness R must not exceed 0.1 μm. (Use the Size/Slenderness chart, Figure 11.31, and the Tolerance/Roughness chart, Figure 11.33. or the *CPS* software if available). (†††)

D6.4 Electron-microscope grids

Small grids of copper (melting point = 1360 K; hardness = 160 MPa; density = 8.96 Mg/m^3) are required to support samples for electron microscopy. The grids are circular disks, 5 mm in diameter and 0.2 mm thick, pierced by a grid of small apertures. A tolerance better than ±0.03 mm, and a surface roughness less than 0.5 μm is required. Use the Size/Slenderness chart, Figure 11.31, and the Tolerance/Roughness chart, Figure 11.33. or the *CPS* software if available. (†††)

D6.5 A granite laser bench

A granite slab is to be used as an optical bench for laser experiments. The top surface is required to be flat to within 0.01 mm, with an RMS roughness of less than 0.2 μm. Can this be achieved by cutting and grinding? (†††)

D6.6 Choosing the cheapest process

A component can be manufactured by machining from the solid, cold forging and cold extrusion. Approximate cost data, in units of the material cost of the component, are given in the table. (Assume that the capital cost of equipment and the cost of power, space and research and development have been absorbed in the overhead rate). Advise on the cheapest process for a batch size of (a) 1, (b) 100 and (c) 10,000. (†††)

Process	Machine	Forge	Extrude
Material cost, C_m	1	1	1
Overhead rate, C_L (hr^{-1})	150	150	150
Tooling cost, C_t	30	3000	9000
Production rate, $\dot{n}$ (hr^{-1})	1	3	15

D6.7 Casting costs

In an analysis of the cost of casting of a small aluminium alloy component, costs were assigned to tooling, overhead and materials in the way shown in the table. The costs are in units of the material cost for the component — in this example, this was £0.04. Identify the cheapest process for a batch size of (a) 100 units, and (b) 10^6 units. (†††)

Process	Sand casting	Investment casting	Pressure die	Gravity die
Material cost, C_m	1	1	1	1
Overhead rate, C_L (hr^{-1})	500	500	500	500
Tooling cost, C_t	50	11 500	25 000	7500
Production rate, $\dot{n}$ (hr^{-1})	20	10	100	40

D7 Use of data sources*

The problems of this section involve data retrieval.

D7.1 Data for cast iron

Use standard data books to retrieve the density, modulus, strength and fracture toughness of plain cast iron. (Primary references: *The ASM Metals Handbook, Smithells* and the *Chapman and Hall Materials Selector*). Then do the same, using the *CMS* software, if available. (†††)

D7.2 Data for magnesium alloys

The candidate materials for a shaker table, identified in Section 6.16, p. 137, included magnesium alloys. Use standard handbooks, first, to locate a castable magnesium alloy with high damping capacity (above 0.5%), and then to retrieve data for the design-limiting properties: modulus and density (for vibration frequency), loss coefficient, and 0.2% proof strength. (Primary references: *The ASM Metals Handbook, Smithells* and the *Chapman and Hall Materials Selector, CMS* software if available.) (†††)

(DTD 5005 or ASTM HZ32A, C80-63)

D7.3 Data for PTFE and leather

Candidate materials for elastic hinges (Section 6.10, p. 116) include PTFE (polytetrafluorethylene, Teflon) and leather. Use standard data books to locate information — as far as you can — for the density, modulus, strength, cost of these. (Primary references: *Materials Engineering Materials Selector, Chapman and Hall Materials Selector*, and the *Handbook of Polymers and Elastomers*.) Then do the same, using the PLASCAMS or the *CMS* software, if available. (†††)

D7.4 Thermal properties of oak

Use standard sources to locate the thermal properties of oak: the thermal conductivity, specific heat, density (and hence the thermal diffusivity) and expansion coefficient. Start by reading approximate values from the Charts. Then try the standard handbooks. Finally, turn to the *CMS* software, if available. (†††)

D7.5 Thermal properties of granite

Granite is used for optical benches and as a stable support for precision metrology. One measure of the suitability of a material for precision instrumentation was (Section 6.20, p. 151) the index

$$M = \lambda/\alpha$$

where λ is the thermal conductivity and α the thermal expansion coefficient. Locate data for these two quantities, for granite (Handbooks first, *CMS* software second). (†††)

* All references to handbooks and databases are detailed in Chapter 13, Appendix 13A.

D7.6 Data for silicon carbide

Weight is important in choosing materials for the rotor of a turbocharger: a light rotor speeds up faster, making the charger more responsive to the driver's needs. It is suggested that weight could be saved by making the rotor out of silicon carbide instead of steel. Write a brief report (along the lines of the Case Studies of Chapter 14) detailing the mechanical, thermal, wear and processing properties of silicon carbide, drawing on standard data sources. Comment on the reliability of the data. (Primary sources: Morrell: *Handbook of Properties of Technical and Engineering Ceramics*, *ASM Engineered Materials Reference Book*, and the *Handbook of Ceramics and Composites; CMS* software if available.) (†††)

D8 Material optimization and scale

The problems of this section illustrate selection when a length-scale is involved.

D8.1 Scaling law for nails

Nails come in a range of sizes as shown in Figure D8.1. When their diameter is plotted against their length on log scales, as shown in the lower part of the figure, the points fall on a line of slope 2/3. This means that nails do not scale isometrically: long nails are relatively thinner than short ones.

(a) Develop a model for the design of a nail, assuming that the length, for a given diameter, is limited by elastic buckling, and that the force required to drive a nail into wood is proportional to the diameter of the nail — this is found, by experiment, to be so. Use your model to explain the scaling law revealed by Figure D8.1. Use data from the figure to determine the value of any unknown constants in your model.

(b) Use your results to explain why copper nails are fatter than steel ones of the same length. Explore whether the model will account accurately for the separation of the lines describing copper and steel nails.

(c) It is required to produce a range of aluminium alloy nails. Using your model, calculate the appropriate diameter for an aluminium nail of length 50 mm.

(d) Comment on the general problem of the way in which material choice affects proportion, and vice versa.

(Young's modulus, E, for steel, is 210 GPa; for copper it is 124 GPa; for aluminium and its alloys it is 71 GPa.) (†††††)

D8.2 Optimizing heat treatment of steel

The drum of a high-speed centrifuge consists of a thin-walled cylinder of radius 1 m which spins about the cylinder axis. The principal loading is that due to centrifugal forces acting on the material of the drum, which may contain internal crack-like defects with a size of up to 2 mm (the resolution limit of the inspection procedures). It is required to maximise the angular velocity of the centrifuge, but cost must also be taken into consideration.

(i) Outline briefly the steps involved in selecting a material for the drum.

(ii) Given a free choice, identify potential candidate-materials, using the Materials Selection Charts as appropriate.

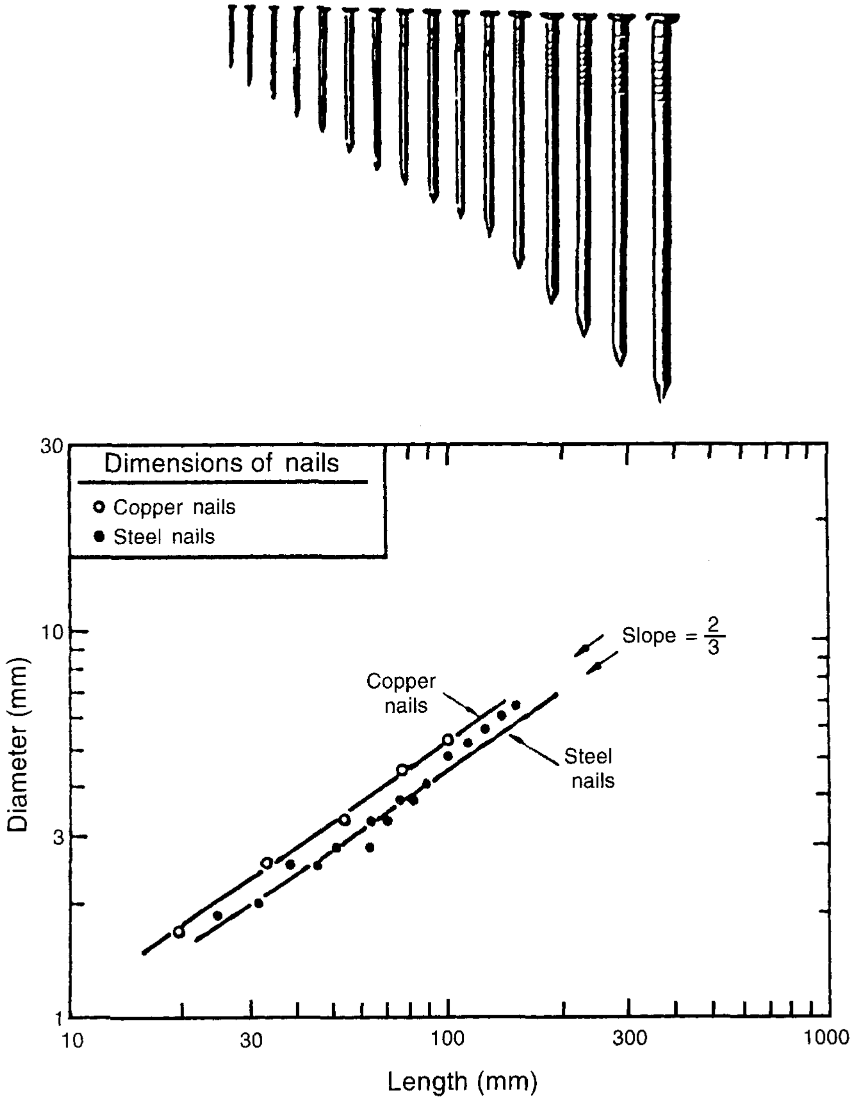


Fig. D8.1 A family of nails (above) and the way their diameter varies with length.

(iii) A heat-treatable steel (density 7.9 Mg/m^3) is selected as a candidate for further investigation. Heat treatment changes both the yield strength and the fracture toughness; the two are related by the equation

$$K_{Ic} = \frac{60\,000}{300 + \sigma_y}$$

where σ_y is in units of MPa and K_{Ic} is in units of MPa m$^{1/2}$. Assuming that the steel contains no defect larger than 2 mm in diameter, calculate the optimum yield strength to be obtained by heat treatment of the steel and hence find the maximum safe angular velocity. (†††††)

D8.3 Optimizing fibre fraction in composites

You have been asked to evaluate the design of the main cross-beam which supports the mast of a new class of ocean racing catamaran. It must be as light as possible.

(a) The manufacturers have suggested using a glass fibre reinforced plastic (GFRP) beam of length, L, equal to the overall beam of the catamaran, and width, w. They have considered it to be simply supported at each end and loaded by the mast at the centre. The beam must have a stiffness greater than or equal to S. The volume fraction, f, of glass fibres (laid unidirectionally) can be varied between 0 and 0.6, and the depth, d, of the beam varied to produce the best combination of properties. The Young's modulus and the density of the composite depend linearly on the volume fraction of the glass fibres. Ignoring self-weight, show that the optimum fraction of glass in the beam for minimum weight is:

$$f = \frac{\rho_m}{2(\rho_g - \rho_m)} - \frac{3E_m}{2(E_g - E_m)}$$

where ρ_m, E_m are the density and modulus of the epoxy matrix polymer, and ρ_g, E_g are those of the glass. Can the optimum for this shape be fabricated?

(b) Briefly, discuss the considerations that influence the choice of production method for this beam, with reference to precision, tolerance, surface finish, batch size, and joining. (†††††)